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Generalized Chaplygin Gas Dominated Anisotropic Bianchi Type-I Cosmological Models

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Abstract We present Bianchi type-I cosmological models in the presence of generalized Chaplygin gas and perfect fluid for early and late time epochs. Exact solutions of Einstein's field equations for this model are obtained. The general solutions of gravitational field equations are expressed in an exact parametric form, with average scale factor as parameter. In the limiting cases of small and large values of the average scale factor, the solutions of the field equations are expressed in exact analytic forms. Moreover, this model predicts that the expansion of Universe is accelerating for the late times. The physical and geometrical properties of the corresponding cosmological models are discussed.

Keywords Cosmological model · Chaplygin gas model · Einstein's field equations · Dark energy and dark matter

1 Introduction

The astronomical observations of the luminosity-distance and redshift relations of type Ia supernovae [1, 2], Cosmic microwave background radiation (CMBR) [3], the galaxy power spectrum [4] etc. indicate that the expansion of the universe is accelerating. This appears to be in strong disagreement with the standard picture of a matter dominated universe.

These observations can be accommodated theoretically by postulating that certain exotic matter with negative pressure dominates the present epoch of our universe. Such exotic matter, called Quintessence, behaves like a vacuum field energy with repulsive (anti-

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gravitational) character arising from the negative pressure. Initially some researchers attribute the observed acceleration to a possible breakdown of our understanding of the laws of gravitation, thus they attempted to modify the Friedmann equation [5–14]. However many more believed that the cosmic acceleration is driven by an exotic energy component called dark energy. Finally it is concluded that, in the frame work of general relativity, about two third of the total energy density of the universe consists of dark energy which is still an unknown component with relativistic negative pressure $p < -\rho/3$ [15]. There are different candidates to play the role of dark energy such as the cosmological constant Λ [16–18], quintessence [19–21], phantom [22–24], quintom [25, 26] etc.

The transition from a universe filled with matter to an exponentially expanding universe does not necessarily require the presence of a scalar field as the only alternative. It would be important if a unified dark matter-dark energy scenario could be found, in which these two components are different manifestations of a single fluid.

Negative pressure leading to an accelerating universe can also be obtained in a Chaplygin gas cosmology [27, 28], in which the matter is taken to be a perfect fluid obeying an exotic equation of state. This cosmological model has some interesting properties. In particular, the Chaplygin gas behaves as pressure less fluid for small values of scale factor and as a cosmological constant for large values of scale factor which tends to accelerate the expansion.

The generalized Chaplygin gas (GCG) [28] is a recent proposal in order to explain the observed acceleration of the universe. This exotic fluid has been considered as an alternative to quintessence and to the cosmological constant, which are other serious candidates to explain the accelerated expansion of the universe. Many observational constraints have been obtained for cosmological models based on the GCG [29]. This model gives the cosmological evolution from the initial dust-like matter to an asymptotic cosmological constant with an epoch that can be seen as a mixture of a cosmological constant and a fluid obeying an equation of state $p = \omega\rho$. The effect of the Chaplygin gas on the dynamics of the universe is dominant during later stages of the matter dominated epoch.

Several authors have studied various aspects of Chaplygin gas and further used it to obtain different cosmological models. Debnath et al. [30] have analyzed the flat Friedmann model filled with Chaplygin fluid in terms of the recently proposed state finder parameters. Wu et al. [24] have examined observational constants on the generalized Chaplygin gas model for dark energy from the Hubble parameter versus redshift data. Cruz et al. [31] have considered the generalized Chaplygin gas as a model for dark energy due to its dark energy like evolution at late times. Setare [32] has taken correspondence between the holographic dark energy density and Chaplygin gas energy density in FRW universe. Guo and Zhang et al. [33] have shown a new generalized Chaplygin gas model that includes the original Chaplygin gas model as a special case.

In this paper, we analyze Einstein's field equations for an anisotropic Bianchi type I space time filled with a generalized Chaplygin gas. We study the behavior of this generalized Chaplygin gas and show that at large cosmological scales, it could account for the current observations of the acceleration of the universe. Physically realistic exact solutions are obtained for small and large values of the average scale factor. The physical and kinematical properties of the models are also discussed.

2 Model and Field Equations

We consider the line element of a Bianchi type I space-time, which is a subsequent generalization of the flat Friedmann-Robertson- Walker (FRW) metric to the anisotropic case. In the co moving coordinates ($u^i = \delta^i_0$), the line element is given as

$$ds^2 = dt^2 - X^2(t)dx^2 - Y^2(t)dy^2 - Z^2(t)dz^2, \tag{1}$$

where X, Y and Z are metric functions.

The stress energy-momentum tensor in the presence of perfect fluid is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{2}$$

where ρ and p are respectively, the energy density and isotropic pressure. u_i is fluid velocity vector satisfying $g_{ij}u^i u^j = 1$.

For the metric (1) and the energy momentum tensor (2), Einstein's field equations

$$R_{ij} - \frac{1}{2}g_{ij}R = -8\pi G T_{ij}, \tag{3}$$

yield the following independent equations

$$\frac{\ddot{X}}{X} + \frac{\ddot{Y}}{Y} + \frac{\dot{X}\dot{Y}}{XY} = -8\pi G p, \tag{4}$$

$$\frac{\ddot{Y}}{Y} + \frac{\ddot{Z}}{Z} + \frac{\dot{Y}\dot{Z}}{YZ} = -8\pi G p, \tag{5}$$

$$\frac{\ddot{Z}}{Z} + \frac{\ddot{X}}{X} + \frac{\dot{Z}\dot{X}}{ZX} = -8\pi G p, \tag{6}$$

$$\frac{\dot{X}\dot{Y}}{XY} + \frac{\dot{Y}\dot{Z}}{YZ} + \frac{\dot{Z}\dot{X}}{ZX} = 8\pi G \rho, \tag{7}$$

where over-dot means differentiation with respect to cosmic time t. The conservation law for the energy momentum tensor $T^{ij}_{;j} = 0$ gives

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right) = 0. \tag{8}$$

Many authors have tried to find the exact solutions of the field equations (4)–(7) by using different techniques such as generating function by Hajj-Boutros [34], Shri Ram [35] and Camci et al. [36]. Berman [37], and Berman and Gomide [38] have discussed a special law of variation of the Hubble's parameter in cosmological models that yields a constant value of deceleration parameter. However, we are interested here to solve field equations (4)–(7) by using the equation of state for a generalized Chaplygin gas which gives the accelerating universe at the latter stage of its evolution.

A generalized Chaplygin gas (GCG) is a perfect fluid with a polytropic equation of state [28, 39, 40]

$$p = -\frac{B}{\rho^\alpha}, \tag{9}$$

where α is a parameter between 0 and 1, and B is a positive constant. A generalized Chaplygin gas is a phenomenological extension of the Chaplygin gas, which exhausts all the possibilities for a polytropic perfect fluid dark energy candidate, where perturbations are stable on small scales [41]. The behavior in time of a GCG interpolates between dust and a cosmological constant, with an intermediate behavior as $p = \omega\rho$.

Now, we define the following physical and geometrical parameters to be used in solving the field equations (4)–(7).

The average scale factor a of Bianchi type-I is defined as

$$R = \sqrt{-g} = (XYZ)^{\frac{1}{3}}. \tag{10}$$

We defined the generalized mean Hubble's parameter H as

$$H = \frac{\dot{R}}{R} = \frac{1}{3}(H_1 + H_2 + H_3), \tag{11}$$

where $H_1 = \frac{\dot{X}}{X}$, $H_2 = \frac{\dot{Y}}{Y}$, $H_3 = \frac{\dot{Z}}{Z}$ are the directional Hubble's parameters in the directions of x, y and z respectively.

The physical quantities of observational interest in cosmology are the expansion scalar θ, the average anisotropy parameter A and the shear scalar σ^2. All are defined according to

$$\theta = u^i_{;i} = \left(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right), \tag{12}$$

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{13}$$

where $\Delta H_i = H_i - H$, $(i = 1, 2, 3)$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left[\frac{\dot{X}^2}{X^2} + \frac{\dot{Y}^2}{Y^2} + \frac{\dot{Z}^2}{Z^2} - \left(\frac{\dot{X}\dot{Y}}{XY} + \frac{\dot{Y}\dot{Z}}{YZ} + \frac{\dot{Z}\dot{X}}{ZX}\right)\right], \tag{14}$$

where

$$\sigma_1^1 = \frac{4}{3}\frac{\dot{X}}{X} - \frac{2}{3}\left(\frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right), \tag{15}$$

$$\sigma_2^2 = \frac{4}{3}\frac{\dot{Y}}{Y} - \frac{2}{3}\left(\frac{\dot{Z}}{Z} + \frac{\dot{X}}{X}\right), \tag{16}$$

$$\sigma_3^3 = \frac{4}{3}\frac{\dot{Z}}{Z} - \frac{2}{3}\left(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y}\right). \tag{17}$$

From (10) and (11), we have

$$H = \frac{\dot{R}}{R} = \frac{1}{3}\left(\frac{\dot{X}}{X} + \frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right). \tag{18}$$

An important observational quantity is the deceleration parameter

$$q = -\frac{\ddot{R}R}{\dot{R}^2}. \tag{19}$$

The sign of the deceleration parameter indicates whether the model inflates or not. The positive sign of q corresponds to standard decelerating models, whereas the negative sign indicates inflation.

We follow the method used by Saha and Rikhvitsky [42] to solve field equations (4)–(7). Subtracting (5) from (4), one can find

$$\frac{d}{dt}\left(\frac{\dot{X}}{X}-\frac{\dot{Y}}{Y}\right)+\left(\frac{\dot{X}}{X}-\frac{\dot{Y}}{Y}\right)\left(\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right)=0. \tag{20}$$

Using (18) in (20) and integrating, we get

$$\frac{\dot{X}}{X}-\frac{\dot{Y}}{Y}=\frac{k_1}{R^3}. \tag{21}$$

Analogically, we find that

$$\frac{\dot{Y}}{Y}-\frac{\dot{Z}}{Z}=\frac{k_2}{R^3}, \tag{22}$$

$$\frac{\dot{Z}}{Z}-\frac{\dot{X}}{X}=\frac{k_3}{R^3}, \tag{23}$$

where k_1, k_2, k_3 are constant of integrations obeying $k_1+k_2+k_3=0$.

On squaring and adding (21), (22), (23) and using (14), we get σ^2 is proportional to R^{-6} i.e.,

$$\sigma=aR^{-3}, \tag{24}$$

where a is constant. This implies that

$$\frac{\dot{\sigma}}{\sigma}=-\left(\frac{\dot{X}}{X}+\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right)=-3H. \tag{25}$$

From (21)–(23), we can find the values of scale factors X, Y, Z in terms of average scale factors as

$$X=m_1R.\exp\left[\frac{2k_1+k_2}{3}\int\frac{dt}{R^3}\right], \tag{26}$$

$$Y=m_2R.\exp\left[\frac{k_2-k_1}{3}\int\frac{dt}{R^3}\right], \tag{27}$$

$$Z=m_3R.\exp\left[-\frac{k_1+2k_2}{3}\int\frac{dt}{R^3}\right], \tag{28}$$

where m_1, m_2 and m_3 are arbitrary constants obeying $m_1m_2m_3=1$.

Equations (7) and (14)–(18) allow to write the analogue of the Friedmann equation as

$$3H^2=8\pi G\rho+\sigma^2. \tag{29}$$

By using the equation of state of the cosmological matter given by (9), the energy conservation equation (8) can be integrated exactly, thus giving the time evolution of the energy density of the generalized Chaplygin gas filled with Bianchi type-I cosmology as

$$\rho=\left[B+\frac{C}{R^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}}, \tag{30}$$

where C is an arbitrary constant of integration. The effective equation of state in the intermediate regime between the dust dominated phase and the de Sitter phase can be obtained by expanding (30) and (9) in subleading order:

$$\rho \simeq B^{\frac{1}{1+\alpha}} + \left(\frac{1}{1+\alpha}\right)\frac{C}{B^{\frac{\alpha}{1+\alpha}}}R^{-3(1+\alpha)}, \tag{31}$$

$$p \simeq -B^{\frac{1}{1+\alpha}} + \left(\frac{\alpha}{1+\alpha}\right)\frac{C}{B^{\frac{\alpha}{1+\alpha}}}R^{-3(1+\alpha)}. \tag{32}$$

These correspond to the mixture of a cosmological constant $B^{\frac{1}{1+\alpha}}$ and a type of matter described by an equation of state:

$$p = \left(\frac{\alpha}{1+\alpha}\right)\rho. \tag{33}$$

Because $\omega = p/\rho = -B/\rho^{1+\alpha}$, so $B = -\omega_0\rho_0^{1+\alpha}$. Substituting this expression into (30), we get

$$\rho_0 = \left[-\omega_0\rho_0^{1+\alpha} + \frac{C}{R_0^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}}. \tag{34}$$

Equation (34) gives

$$C = \left[\rho_0^{1+\alpha} + \omega_0\rho_0^{1+\alpha}\right]R_0^{3(1+\alpha)}. \tag{35}$$

Again, substituting (35) in to (30), we finally get energy density and pressure, respectively in terms of ω_0 and ρ_0 as

$$\rho = \rho_0\left[-\omega_0 + (1+\omega_0)\left(\frac{R_0}{R}\right)^{3(1+\alpha)}\right]^{\frac{1}{1+\alpha}}, \tag{36}$$

$$p = \omega_0\rho_0\left[-\omega_0 + (1+\omega_0)\left(\frac{R_0}{R}\right)^{3(1+\alpha)}\right]^{-\frac{\alpha}{1+\alpha}}, \tag{37}$$

where a subscript 0 means the value of the variable at the present time. From this result, one can understand a striking property of the GCG. The generalized Chaplygin gas behaves like the cosmological constant when $\omega_0 = -1$ and it behaves like the dust matter when $\omega_0 = 0$. At early times, i.e. the cosmological radius $R(t)$ is small, $\rho \simeq (R_0/R)^3$, which corresponds to a dust like dominated universe. At late times i.e. the cosmological radius is large, $\rho \simeq constant$, which corresponds to a cosmological constant like dominated universe. Therefore the GCG interpolates between a dust dominated phase in the past and a de Sitter phase at late time. This distinct feature makes the model an intriguing candidate for the unification of dark matter and dark energy.

3 Solution of Field Equations

Using (30) into (29) and assuming $8\pi G = c = 1$ onwards, we get the Friedmann equation as

$$3\left(\frac{\dot{R}}{R}\right)^2 = \left[B + \frac{C}{R^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} + \frac{a^2}{R^6}. \tag{38}$$

Due to complicated form of the expression for energy density ρ in (30), it is very difficult to find exact solution for average scale factor $R(t)$ from highly non-linear differential equation (38). Therefore, we try to find the solutions of (38) for small and large values of scale factor in the following subsections.

3.1 Solutions for Small Values of Scale Factor $R(t)$

We see that for a small value of the scale factor $R(t)$, ρ is very large and the solution corresponds to the universe dominated by dust-like matter ($p = 0$). Then from (38), we have

$$3\left(\frac{\dot{R}}{R}\right)^2 = \left(\frac{C}{R^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}} + \frac{a^2}{R^6}. \tag{39}$$

Integrating (39), we get

$$R^3(t) = \frac{3C^{\frac{1}{1+\alpha}}}{4}(t+t_1)^2 - \frac{a^2}{C^{\frac{1}{1+\alpha}}}, \tag{40}$$

where t_1 is the constant of integration. Without loss of any generality, we can take $t_1 = 0$ for simplicity.

Substituting the value of $R(t)$ from (40) into (26)–(28), we get the following explicit solutions for metric functions as

$$X(t) = m_1\left(\frac{3l}{4}t^2 - \frac{a^2}{l}\right)^{1/3}\left[\frac{t-b}{t+b}\right]^{\frac{2k_1+k_2}{a3\sqrt{3}}}, \tag{41}$$

$$Y(t) = m_2\left(\frac{3l}{4}t^2 - \frac{a^2}{l}\right)^{1/3}\left[\frac{t-b}{t+b}\right]^{\frac{k_2-k_1}{a3\sqrt{3}}}, \tag{42}$$

and

$$Z(t) = m_3\left(\frac{3l}{4}t^2 - \frac{a^2}{l}\right)^{1/3}\left[\frac{t-b}{t+b}\right]^{-\frac{k_1+2k_2}{a3\sqrt{3}}}, \tag{43}$$

where $l = C^{\frac{1}{1+\alpha}}$ and $b = \frac{a}{l}\sqrt{\frac{4}{3}}$. The energy density for matter dominated phase is given by

$$\rho(t) = \frac{l}{(\frac{3l}{4}t^2 - \frac{a^2}{l})}. \tag{44}$$

The time evolution of the physical and geometrical quantities in the limit of small values of average scale factor are given by

$$H(t) = \frac{lt}{2[\frac{3l}{4}t^2 - \frac{a^2}{l}]}, \tag{45}$$

$$A = \frac{32b^2({k_1}^2 + {k_2}^2 + k_1k_2)}{27a^2l^2} \cdot \frac{1}{t^2(t^2 - b^2)^2} \left(\frac{3l}{4}t^2 - \frac{a^2}{l}\right)^2, \tag{46}$$

$$\sigma^2(t) = \frac{a^2}{(\frac{3l}{4}t^2 - \frac{a^2}{l})^2}, \tag{47}$$

$$\theta(t) = 3H, \tag{48}$$

$$q = \frac{1}{2} + \frac{2a^2}{l^2t^2}, \tag{49}$$

$$V = R^3. \tag{50}$$

The shear scalar is decreasing function of the cosmic-time, and in the limit $t \to \infty$, corresponding to the isotropic limit, σ^2 tends to zero. We observe that the solutions are of Kasner's type. All the physical parameters such as ρ, H, A, σ^2 and q are decreasing function of the cosmic time. In the limit $t \to 0$, all these parameters tend to infinity and have a singular behavior. In the limit as $t \to \infty$, these parameters tend to zero. The model approaches to isotropy for large value of time. In the limit $t \to 0$, $\sigma^2 \propto V^{-2}$, and the shear scalar has a singular behavior, tending to infinity. In this case the density depending on the co-moving volume is given by $\rho \propto V^{-(1+\alpha)}$. The cosmological evolution is non-inflationary with $q = \frac{1}{2}$ as $t \to \infty$.

3.2 Solution for Large Values of the Scale Factor $R(t)$

For a large value of scale factor $R(t)$, i.e., for late-time evolution, the universe tends to de-Sitter inflationary phase. It is to be noted here that $B^{\frac{1}{1+\alpha}}$ solves the equation $\rho + p = 0$. From (38), we have

$$3\left(\frac{\dot{R}}{R}\right)^2 = B^{\frac{1}{1+\alpha}} + \frac{a^2}{R^6}. \tag{51}$$

Integrating (51), we get

$$R^3 = a \sinh\left[\wedge(t + t_0)\sqrt{3}\right], \tag{52}$$

where t_0 is a constant of integration and $\wedge = B^{\frac{1}{1+\alpha}}$.

Using (52) into (26)–(28), we get the metric coefficients as

$$X(t) = m_1 a^{1/3} \sinh^{1/3}\left[\wedge(t + t_0)\sqrt{3}\right]. \left[\frac{\exp[\wedge(t + t_0)\sqrt{3}] - 1}{\exp[\wedge(t + t_0)\sqrt{3}] + 1}\right]^{\frac{a\wedge(2k_1+k_2)\sqrt{3}}{3}}, \tag{53}$$

$$Y(t) = m_2 a^{1/3} \sinh^{1/3}\left[\wedge(t + t_0)\sqrt{3}\right]. \left[\frac{\exp[\wedge(t + t_0)\sqrt{3}] - 1}{\exp[\wedge(t + t_0)\sqrt{3}] + 1}\right]^{\frac{a\wedge(k_2-k_1)\sqrt{3}}{3}}, \tag{54}$$

and

$$Z(t) = m_3 a^{1/3} \sinh^{1/3}\left[\wedge(t + t_0)\sqrt{3}\right]. \left[\frac{\exp[\wedge(t + t_0)\sqrt{3}] - 1}{\exp[\wedge(t + t_0)\sqrt{3}] + 1}\right]^{\frac{-a\wedge(k_1+2k_2)\sqrt{3}}{3}}. \tag{55}$$

The time evolution of the physical and geometrical quantities in the limit of large value of $R(t)$ are given by

$$H = \left(\frac{\wedge\sqrt{3}}{3}\right).\coth[\wedge(t+t_0)\sqrt{3}], \tag{56}$$

$$A = 24a^2 \wedge^2 ({k_1}^2 + {k_2}^2 + k_1 k_2)\frac{\exp[2\wedge(t+t_0)\sqrt{3}]}{\exp[2\wedge(t+t_0)\sqrt{3}] - 1}\tanh^2[\wedge(t+t_0)\sqrt{3}], \tag{57}$$

$$\sigma^2 = \text{csch}^2[\wedge(t+t_0)\sqrt{3}], \tag{58}$$

$$\theta = \sqrt{(3\wedge)}.\coth[\wedge(t+t_0)\sqrt{3}], \tag{59}$$

$$V = R^3, \tag{60}$$

$$q = -1 + 3\text{sech}^2[\wedge(t+t_0)\sqrt{3}]. \tag{61}$$

4 Statefinder Parameters

In 2003, Sahni et al. [43] proposed a pair of parameters $[r, s]$, called statefinder parameters. The statefinder probes the expansion dynamics of the universe through higher derivatives of the scale factor $\dddot{R}$ and is a natural comparison to the deceleration parameter q which depends upon $\ddot{R}$.

The statefinder is a 'geometrical' diagnostic in the sense that it depends upon the expansion factor and hence upon the metric describing space-time. Trajectories in the $r - s$ plane, corresponding to different cosmological models, exhibit qualitatively different behaviors. The spatially flat ΛCDM scenario corresponds to a fixed point in the diagram $[r, s]$, $\Lambda CDM = [0, 1]$. Departure of a given dark energy model from this fixed point provides a good way of establishing the distance of this model from ΛCDM. In what follows, we will apply statefinder diagnostic to the GCG model in Bianchi type-I model. The statefinder diagnostic pair $[r, s]$ has the following form

$$r = \frac{\dddot{R}}{RH^3} \quad \text{and} \quad s = \frac{r-1}{3(q-\frac{1}{2})}. \tag{62}$$

These parameters allow us to characterize the properties of dark energy. Trajectories in the $[r, s]$ plane corresponding to different cosmological models, correspond to the fixed points $s = 0$, $r = 1$.

For small value of $R(t)$, the expressions for r and s are as follows

$$r = 1 + \frac{12a^2 H}{l^2 t^2}, \tag{63}$$

$$s = 2. \tag{64}$$

As $t \to \infty$, we get $r = 1$ and $s = 2$.

For large value of $R(t)$, the expressions for r and s are given by

$$r = 1 + 9\text{sech}^2[\wedge(t+t_0)\sqrt{3}], \tag{65}$$

$$s = \frac{2\text{sech}^2[\wedge(t+t_0)\sqrt{3}]}{2\text{sech}^2[\wedge(t+t_0)\sqrt{3}] - 1}. \tag{66}$$

From (65) and (66), a relation between r and s in closed form is given by

$$s = \frac{2(r-1)}{2r-11}. \tag{67}$$

Equation (67) shows that the trajectories in the $[r, s]$ plane, corresponding to different cosmological models, for example ΛCDM model diagrams, correspond to the fixed point $s = 0$ for $r = 1$.

5 Conclusion

We have considered an exotic fluid and have studied its cosmological implications in Bianchi type-I models. Such an exotic fluid obeys an equation of state, we call it generalized Chaplygin gas equation of state, which has interesting properties and provides a phenomenological mechanism of unifying dark matter and dark energy. We have presented generalized Chaplygin gas dominated Bianchi type-I cosmological models with perfect fluid as the matter source. We have solved Einstein's field equations for small and large values of the average scale factor. It has been shown here that this generalized Chaplygin gas of state describes the evolution of a universe from a phase dominated by an equation of state $p = \omega\rho$ for small values of the scale factor to a phase dominated by a cosmological constant $B^{\frac{1}{1+\alpha}}$ for large values of the scale factor. We are able to describe the universe from early universe dominated by a dust like matter to de-Sitter inflationary era. The deceleration parameter is found to be positive in the early phase of matter dominated era which is crucial for the successful nucleosynthesis as well as for the structure formation of the universe. This is consistent with the observational fact that beyond a certain value of the redshift z ($z \approx 1.5$), the universe surely had a deceleration phase of expansion. The effect of dark energy component such as pure Chaplygin gas on the dynamics of the universe is dominated only during later stages of the matter dominated epoch. Ultimately we get the deceleration parameter as negative for large value of the scale factor, which supports that, due to the dark energy component, the present universe is accelerating. We have then discussed this generalized Chaplygin gas with the cosmological evolution by finding the state finder parameters to derive the accelerated expansion of the universe. The statefinder trajectories commences its evolution from $s = 2$ and $r = 1$ and ends it at the ΛCDM fixed point $(s = 0, r = 1)$ in the future.

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